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# Bayesian Inference of Disease Epidemics

## *Smallpox Outbreaks & Public Policy*

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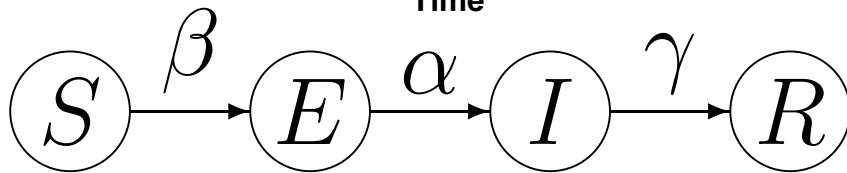
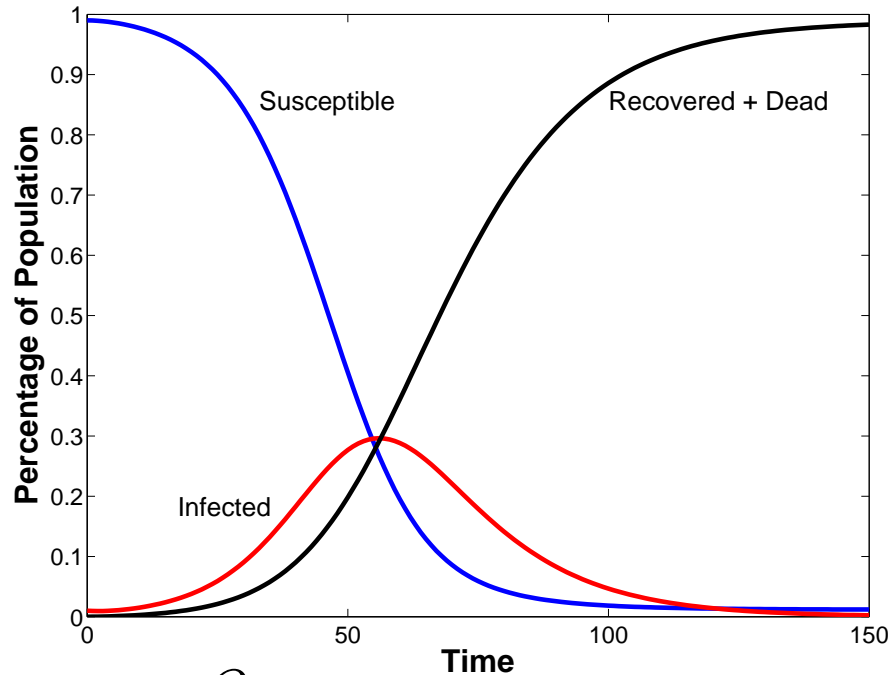
University of Chicago

# Emerging Diseases

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- Public concern over health issues
  - Smallpox, SARS, & Avian Influenza
  - Proper vaccination strategy
- Past models of smallpox dynamics
  - Verbal scenarios
  - Quantitative models

# General Disease Dynamics



$$\begin{aligned}\frac{dS}{dt} &= -\beta SI, \\ \frac{dE}{dt} &= \beta SI - \alpha E, \\ \frac{dI}{dt} &= \alpha E - \gamma I, \\ \frac{dR}{dt} &= \gamma I.\end{aligned}$$

where

$$\beta = \frac{R_0 \gamma}{N}$$

# Smallpox & Emerging Diseases

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- Public concern over health issues
  - Smallpox, SARS, & Avian Influenza
  - Proper vaccination strategy
    - Based on  $R_0$
    - Decreasing the # of Susceptibles
- Past models of smallpox dynamics
  - Verbal scenarios
  - Quantitative models
    - Often ad hoc estimates of  $R_0$
    - Unclear relationship between variance in  $R_0$  and model outcomes

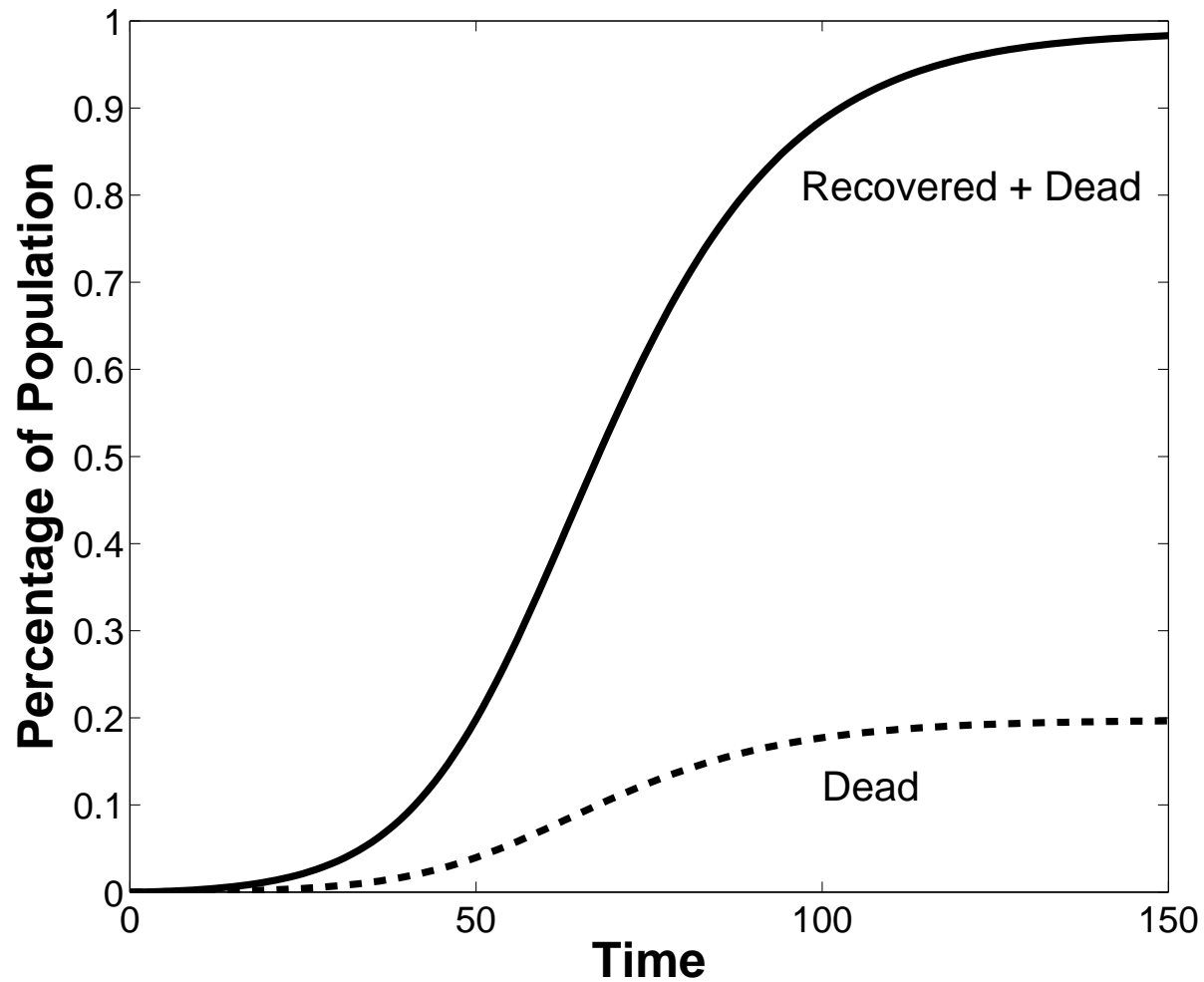
# The Disease Datasets

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- 5 previously exposed populations
  - Prior to widespread vaccination
  - England, New England, & Mauritius
  - Old World
- 8 previously unexposed populations
  - Missions of Southwest & Baja California
  - New World

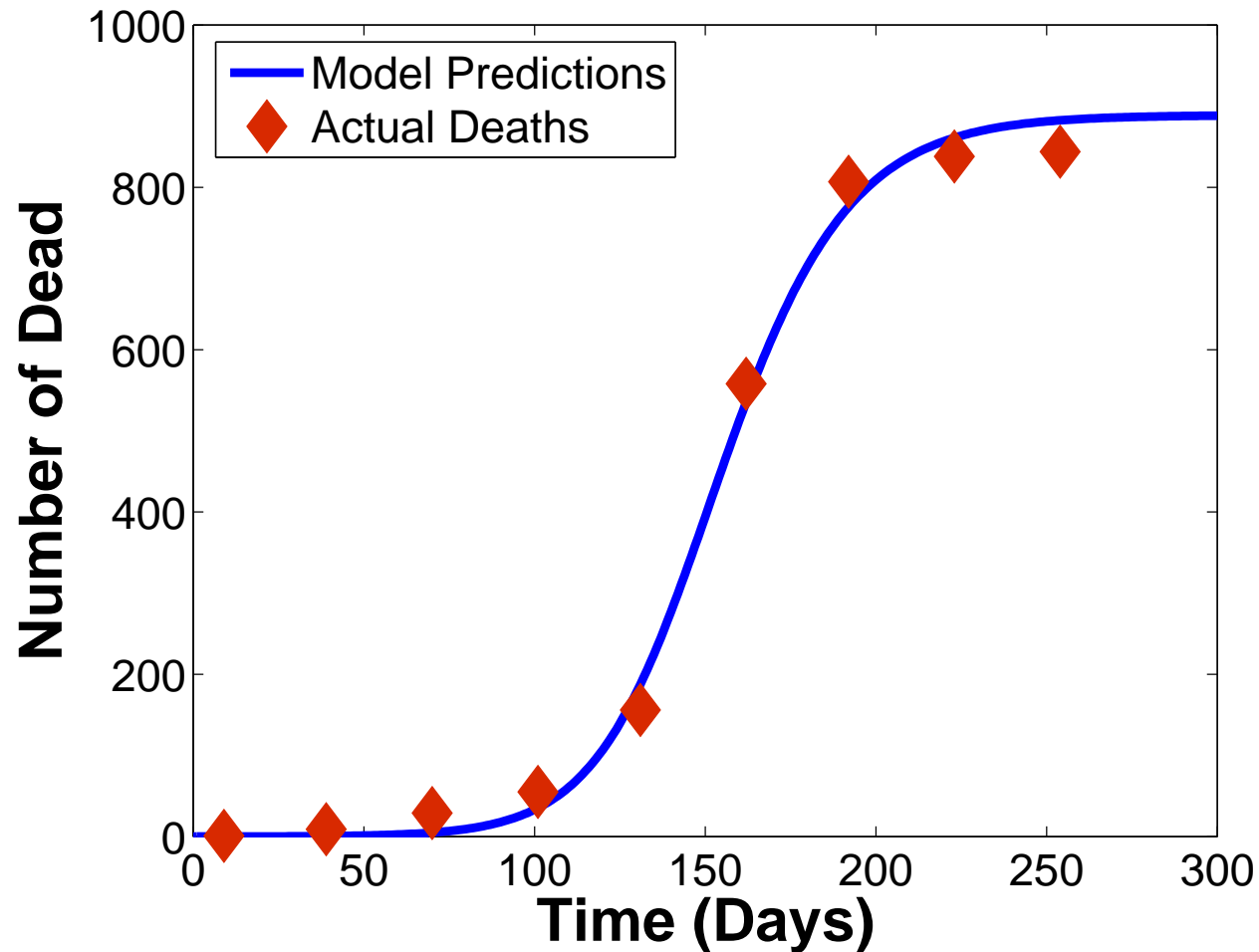
# Smallpox Disease Dynamics

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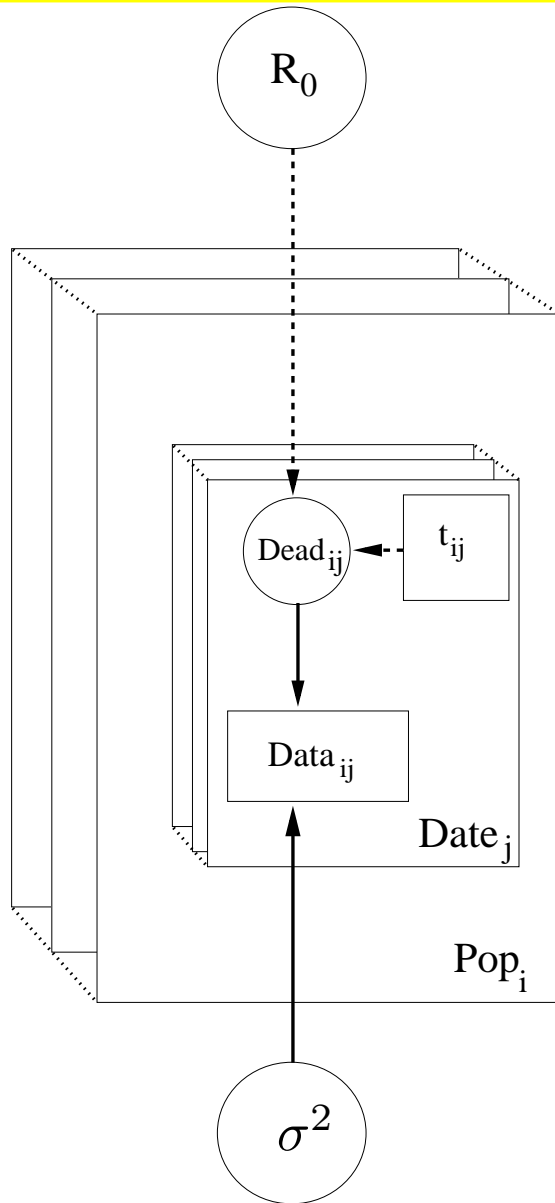


# Smallpox Outbreak - Boston(1721-1722)

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# Bayesian Hierarchical Model



Exposed & Unexposed

$$R_0 \sim \text{LN}(\mu, \sigma_{R_0}^2)$$

$$\sigma^2 \propto 1$$

$$Dead_{ij} = f(R_0)$$

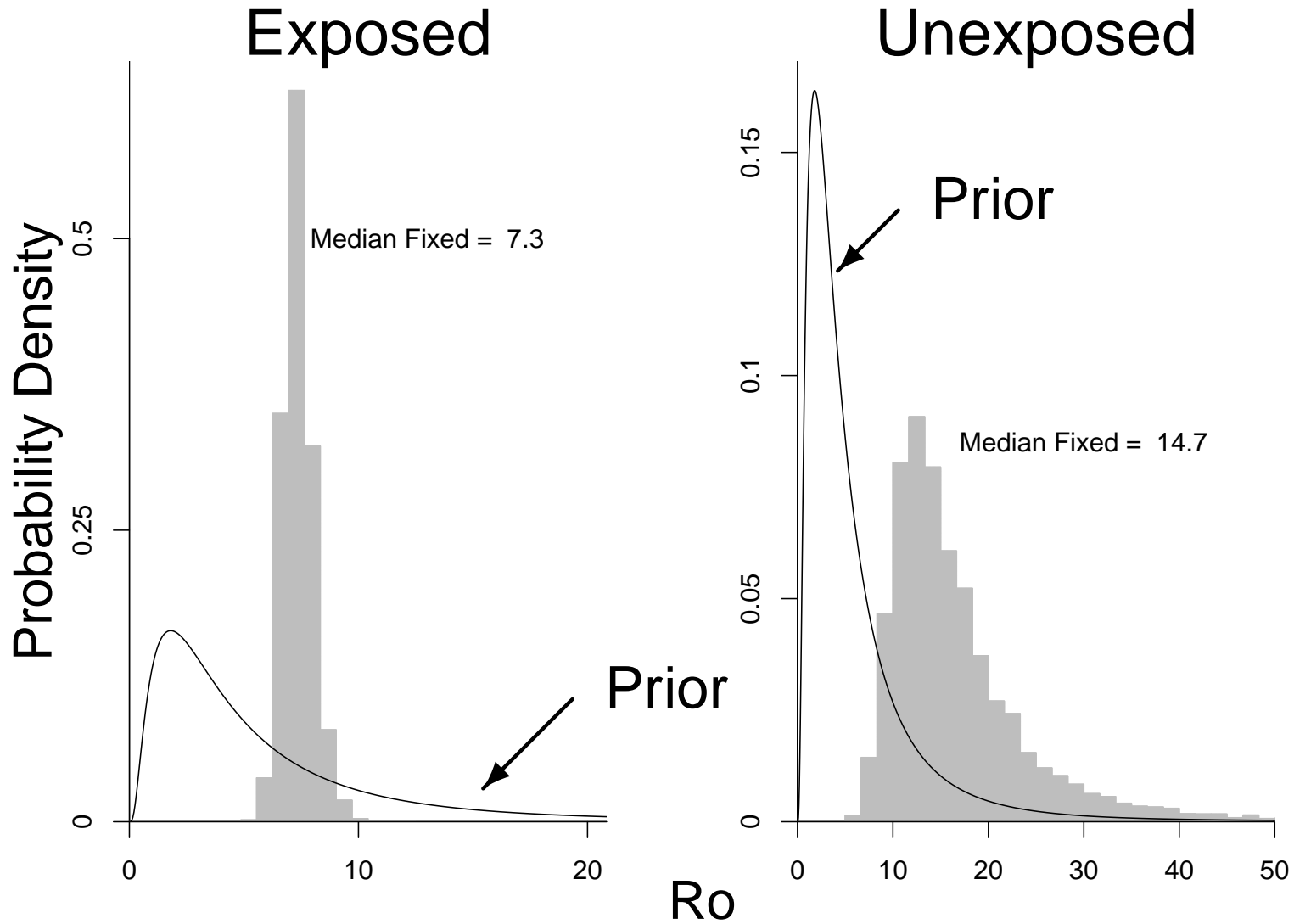
$$Data_{ij} \sim \text{N}(Dead_{ij}, \sigma^2)$$

# Bayesian Analysis Methods

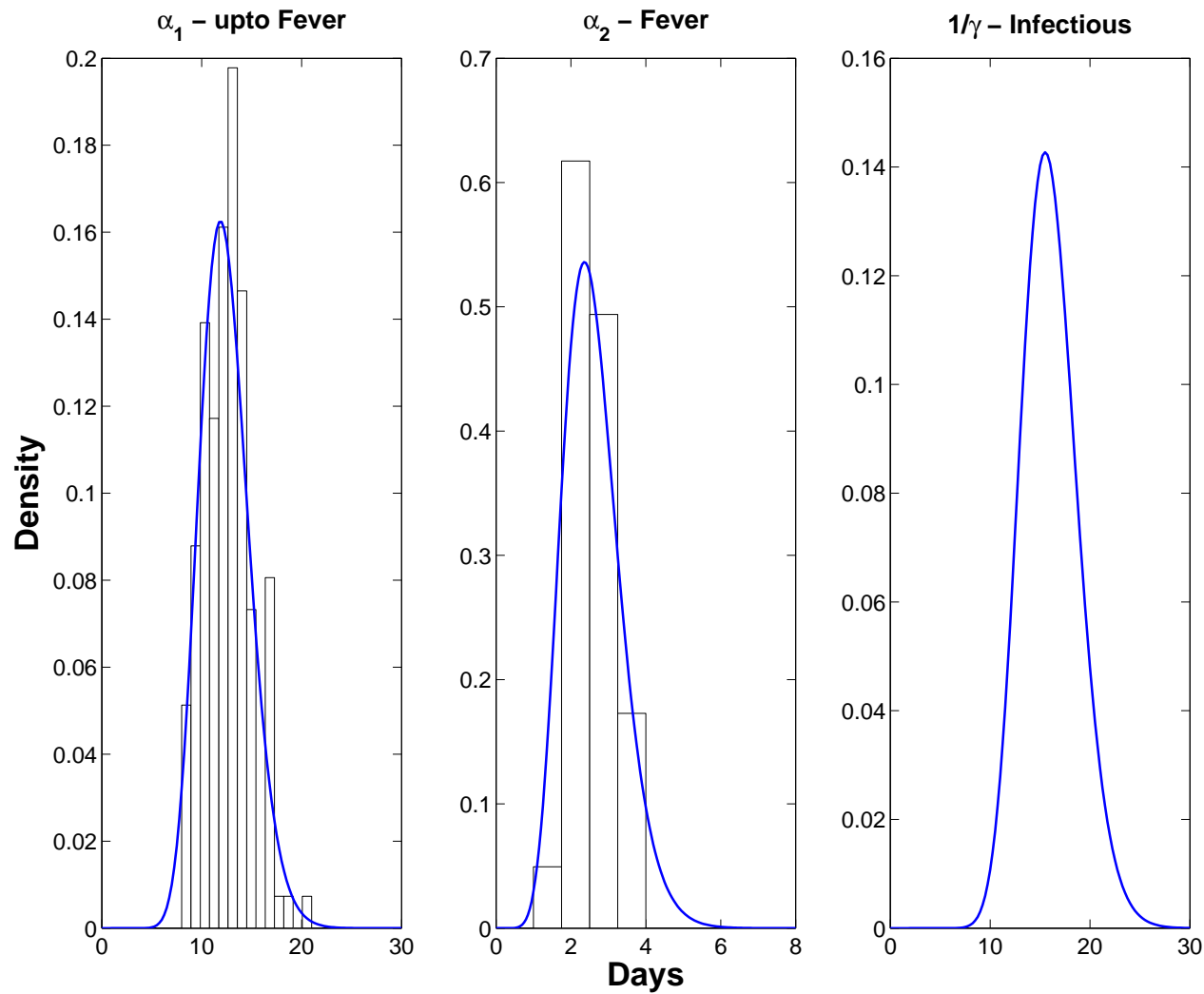
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- Prior on  $R_0$  based on Anderson & May
  - Sets upper bounds for  $R_0$
- Sampling Method: Gibbs with a Metropolis-Hastings step
- Up to six separate chains (n=110,000)
- Removed transients (trimmed), removed autocorrelation (thinned), & tested for convergence
- Chains combined to create the posterior distribution

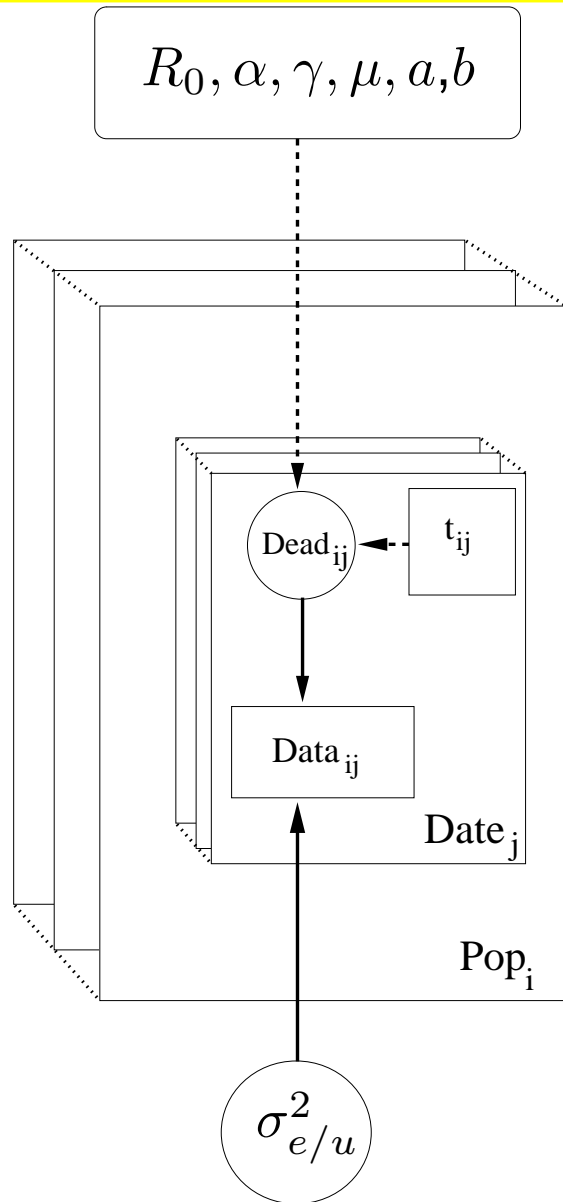
# Varying $R_0$ Only



# Varying other parameters

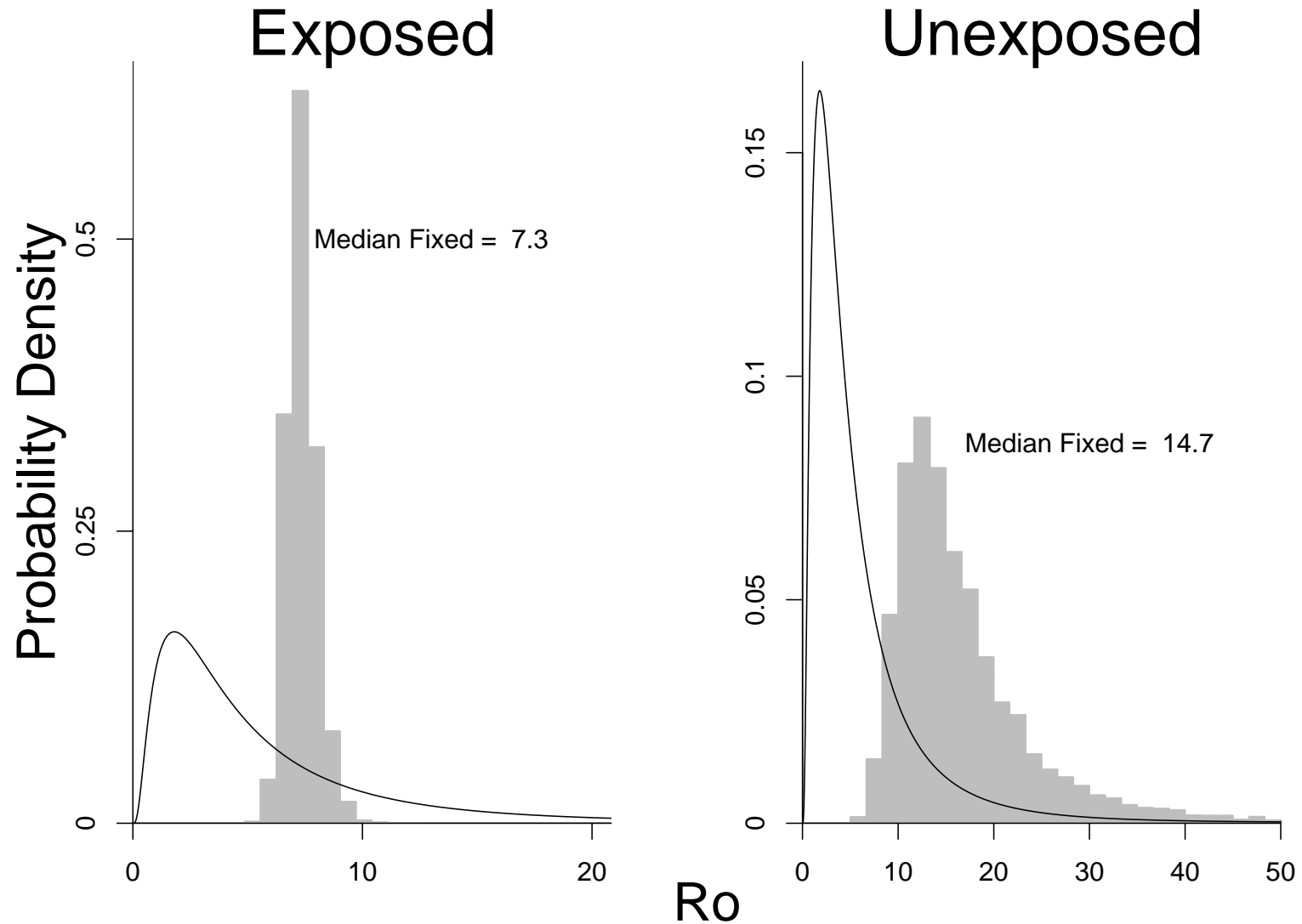


# Bayesian Hierarchical Model – II

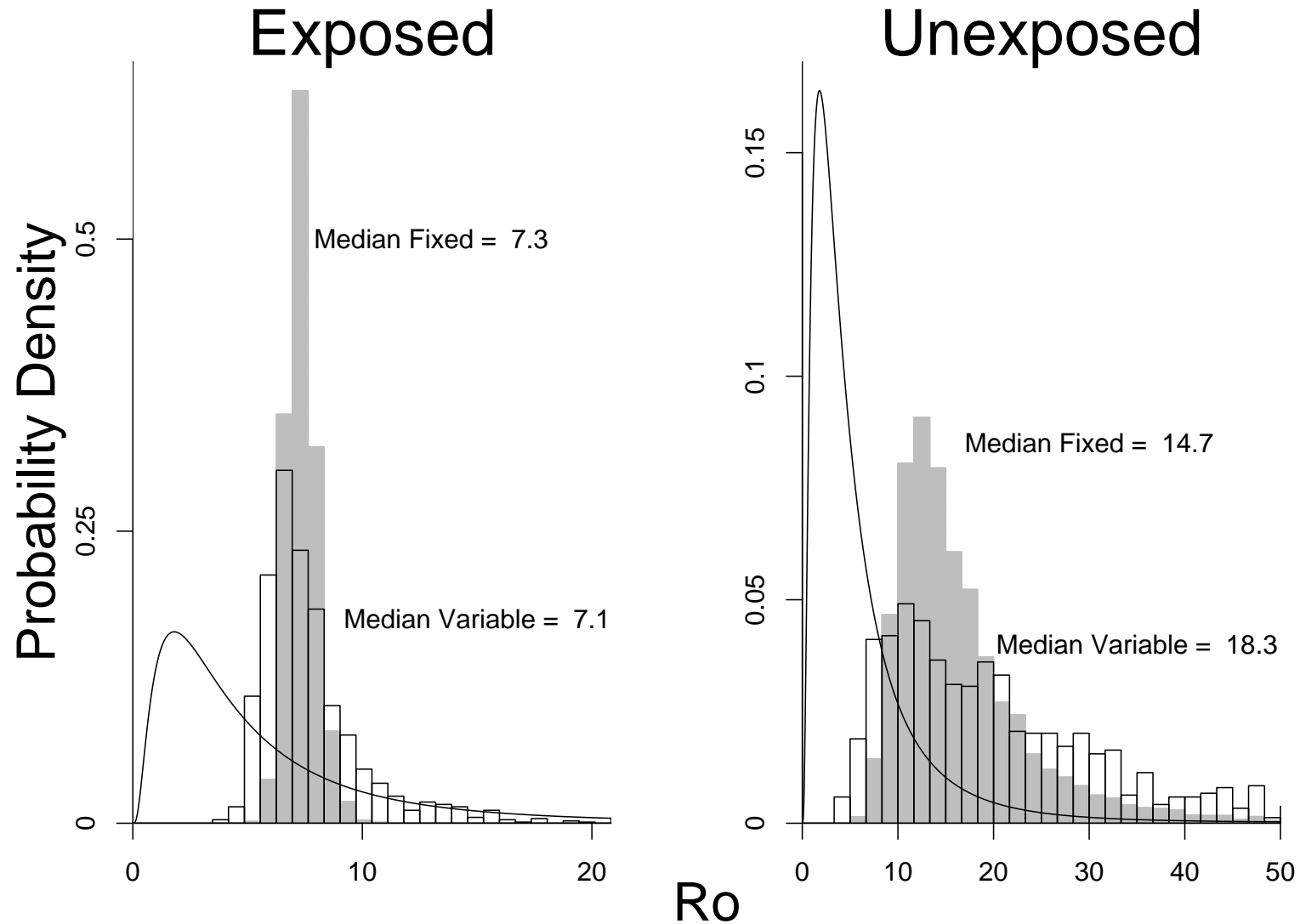


$$\begin{aligned}
 R_{0,e/u} &\sim \text{LN}(m_{e/u}, \sigma_{R_{0,e/u}}^2) \\
 \sigma_{e/u}^2 &\propto 1 \\
 \alpha &= \frac{1}{\alpha_1 + \alpha_2} \\
 \alpha_1 &\sim \Gamma(\text{mean} = 2.6, \text{var} = 0.6) \\
 \alpha_2 &\sim \Gamma(\text{mean} = 12.3, \text{var} = 6.1) \\
 1/\gamma &\sim \Gamma(\text{mean} = 16.0, \text{var} = 8.0) \\
 \mu_{e/u} &\sim \text{Be}(a_{e/u}, b_{e/u}) \\
 \frac{a_e}{a_e + b_e} &\sim \text{Be}(5, 17) \\
 \frac{a_u}{a_u + b_u} &\sim \text{Be}(4, 4.5) \\
 a_e &\sim \Gamma(\text{mean} = 30, \text{var} = 150) \\
 a_u &\sim \Gamma(\text{mean} = 4, \text{var} = 4)
 \end{aligned}$$

# Varying other parameters – $R_0$

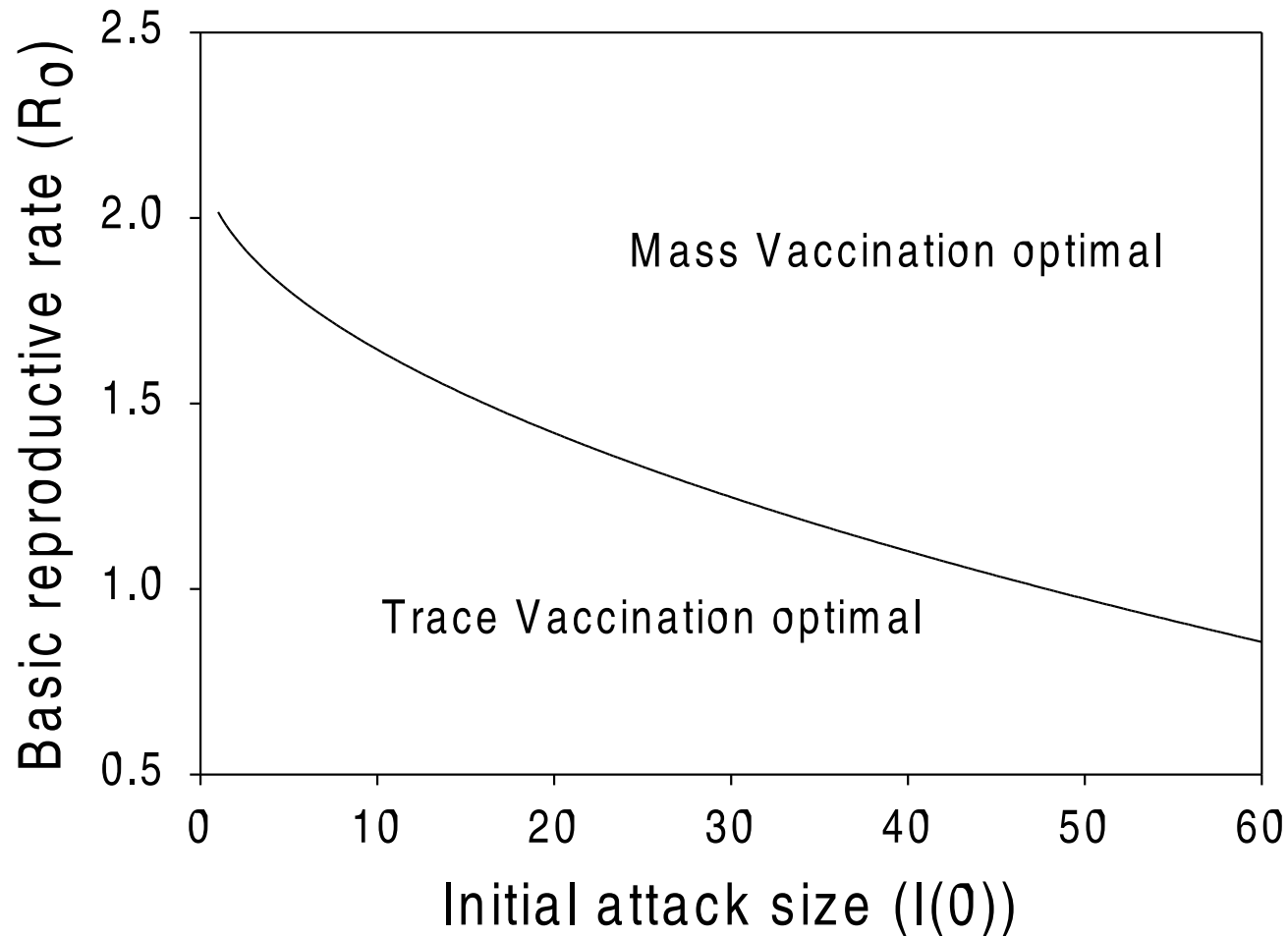


# Varying other parameters – $R_0$



# Smallpox Disease Dynamics

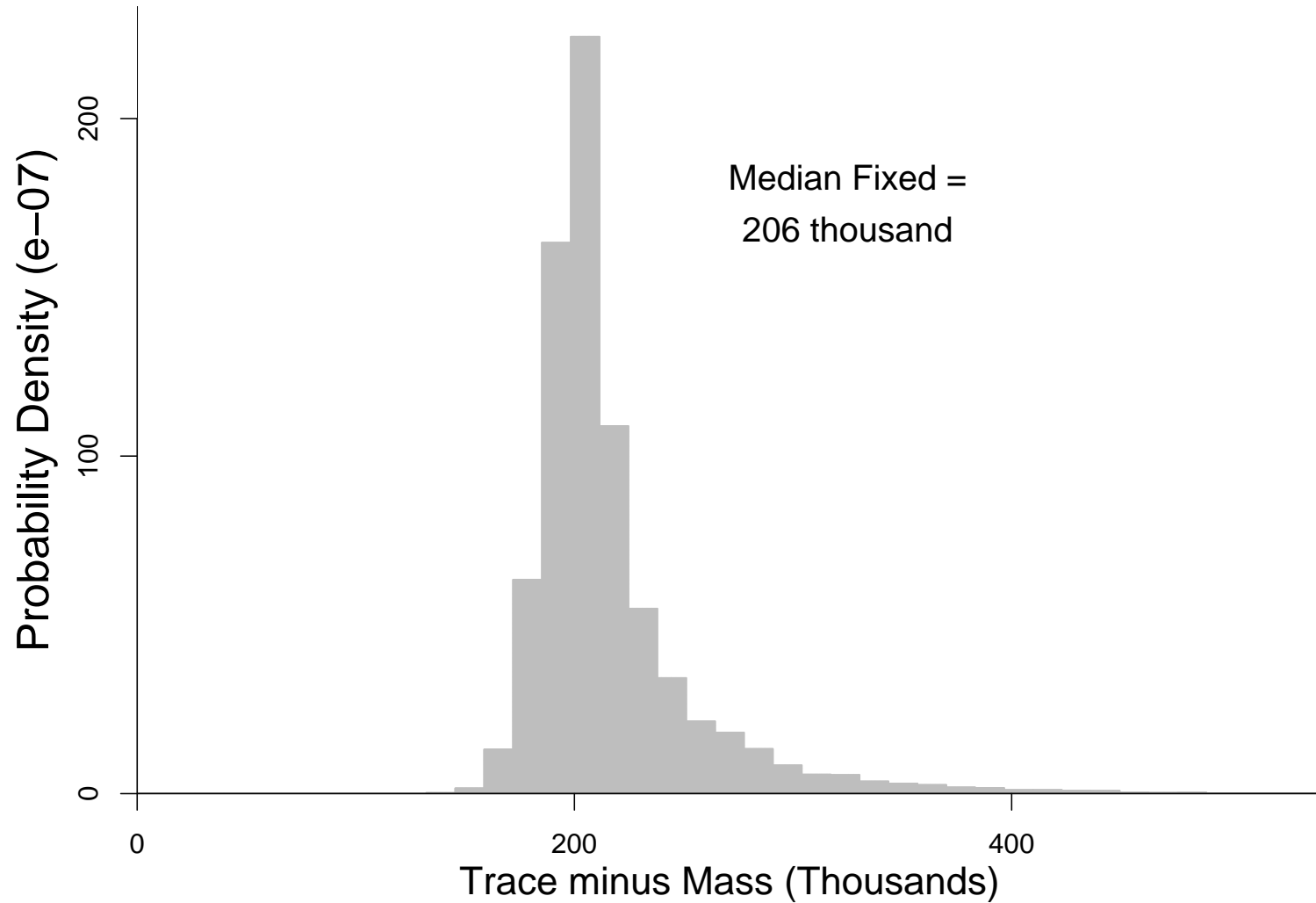
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from Kaplan et al. 2002 PNAS

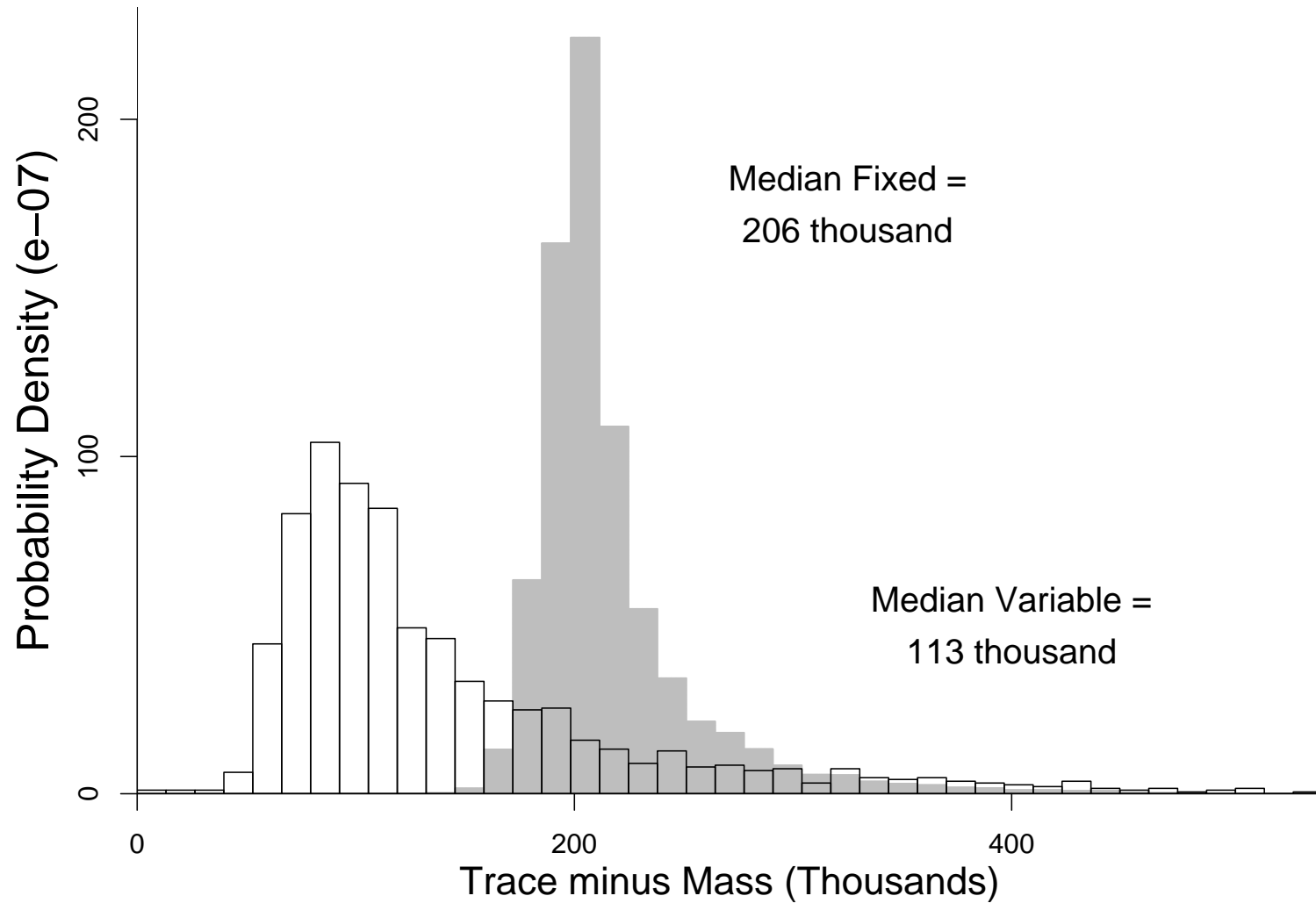
# Exposed Mortalities

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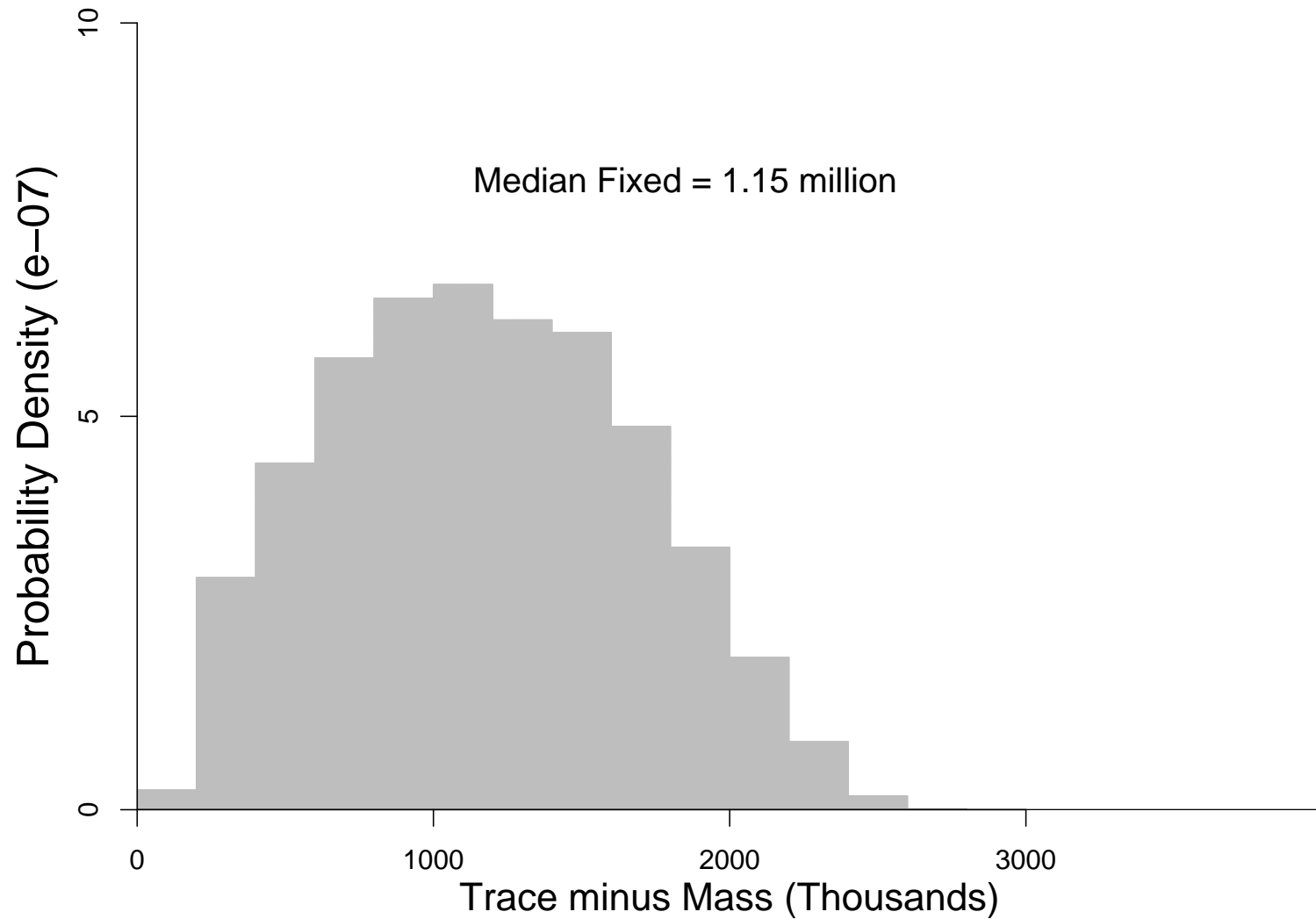
# Exposed Mortalities

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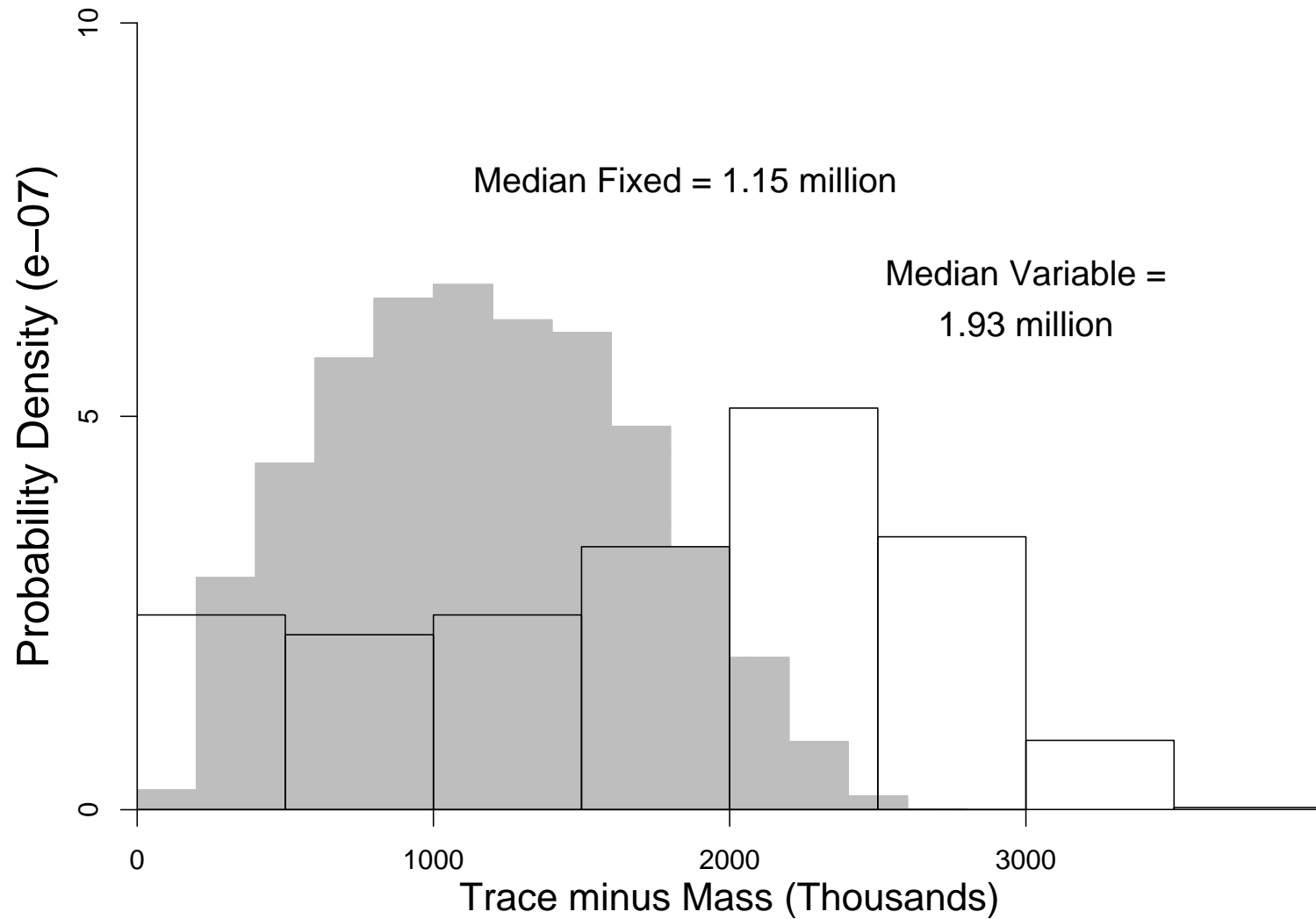
# Unexposed Mortalities

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# Unexposed Mortalities

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# Smallpox – Conclusion

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- Importance of considering variation
  - Ecological models & Public policy often focus on best fit parameters
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  - Ecological models & Public policy often focus on best fit parameters
  - Need to consider a range of values
- Smallpox – Mass vaccination given an outbreak
- Protects against potentially devastating attacks
- Spatial correlation in  $R_0$  for New World
- Apply to other epidemic dynamics
  - Avian Influenza, SARS, ...
  - Gypsy Moth outbreaks

# Acknowledgements & Support

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- Center for Integrating Statistics and Environmental Science
- EPA STAR grant
- Greg Dwyer & Vanja Dukic