Bayesian inverse modeling of "hidden" belowground ecosystem processes

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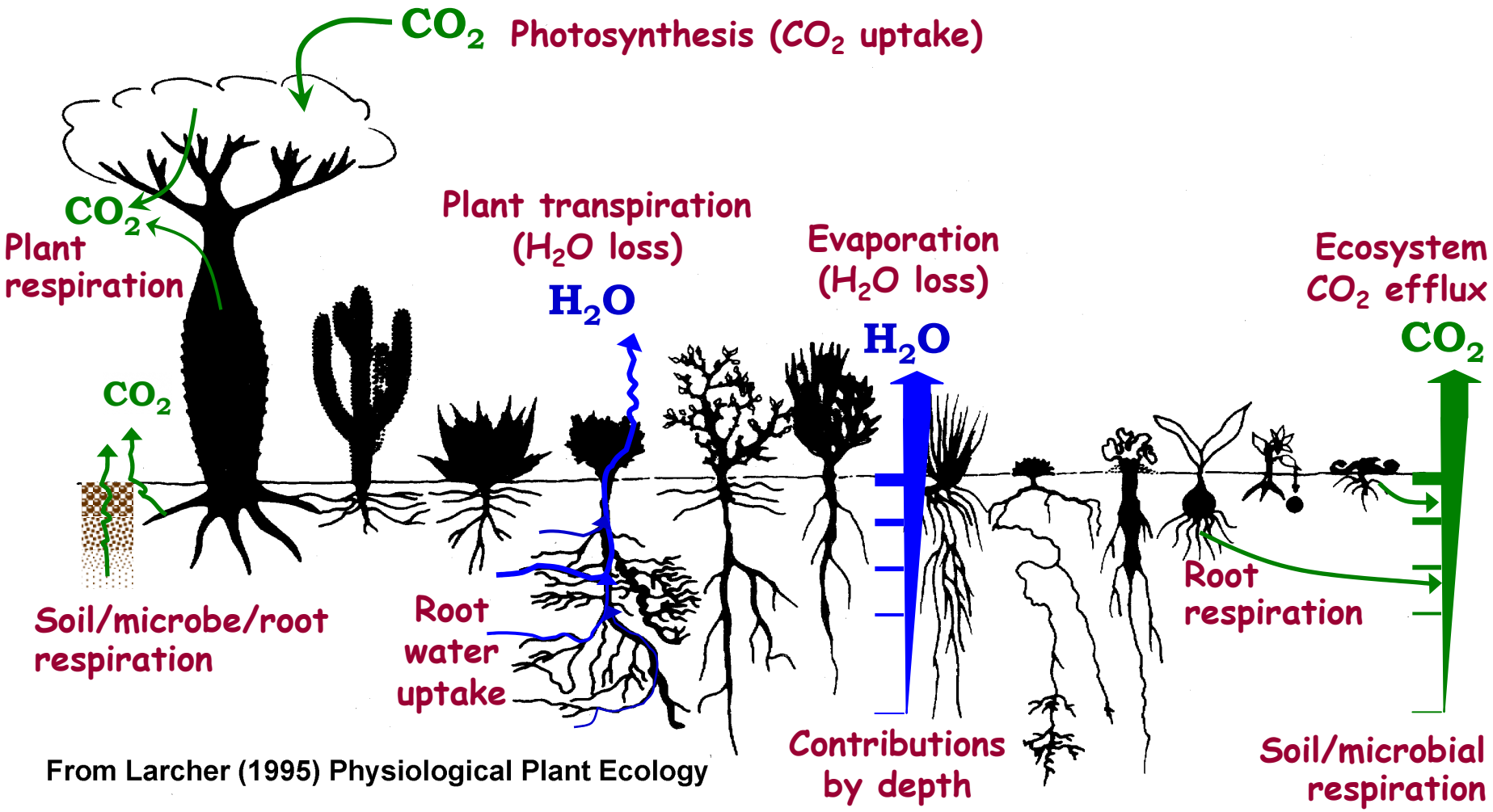
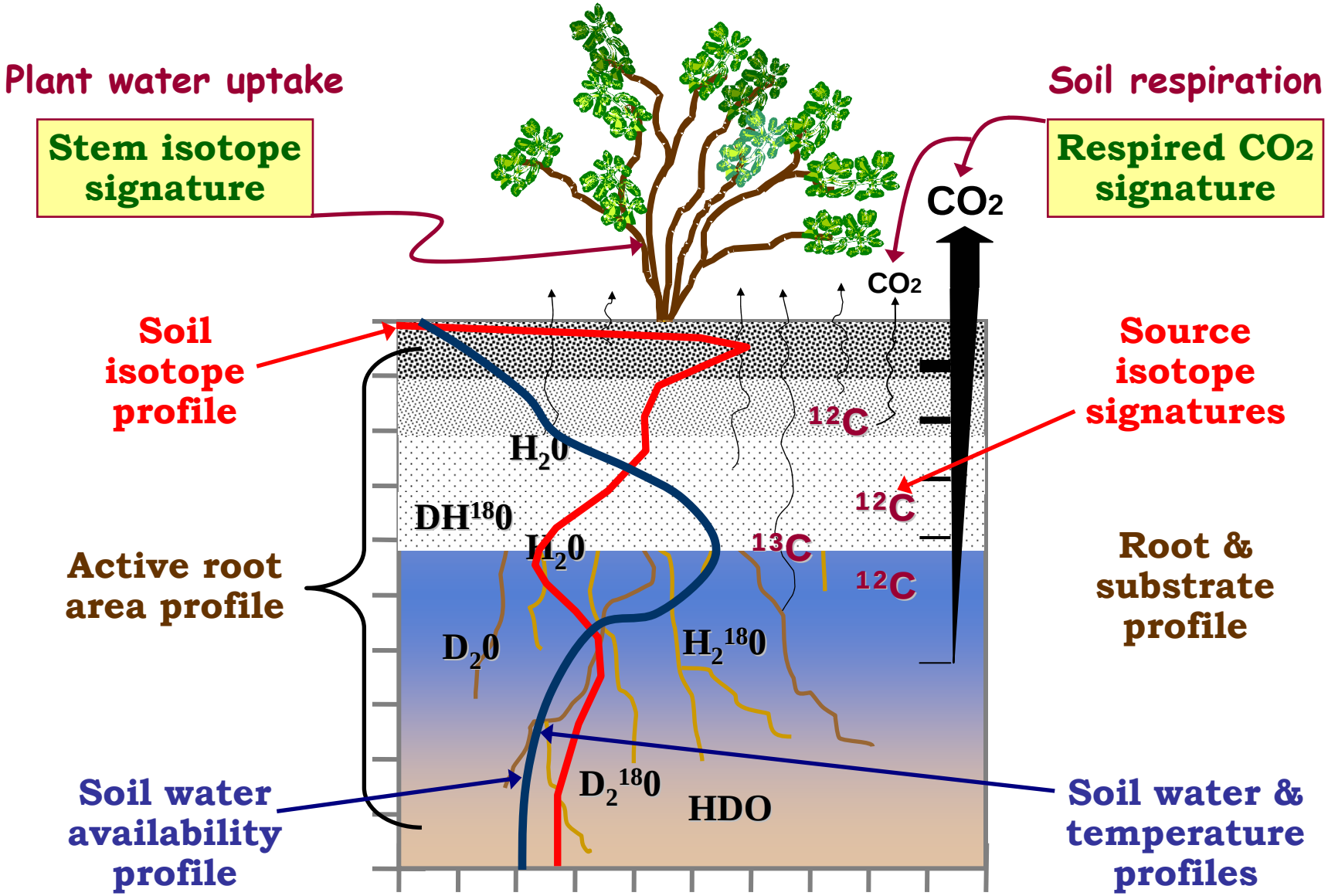
Carbon and Water Fluxes



From Larcher (1995) Physiological Plant Ecology

Stable Isotope Tracers



The Inverse Problem

$$H(t \mid \theta) = \int h(t, z \mid \theta)\, dz$$

low dimension

observable
ABOVEGROUND
process

high dimension

unobservable
BELOWGROUND
process

GOAL: estimate & understand $h(t,z \mid \theta)$ and θ

APPROACH:
Bayesian Hierarchical Inverse Modeling

The Process Models

Plant water uptake

$$\delta D_{stem}(t) = \int_0^B \delta D(z) \cdot q(z,t)\,dz$$

$$\delta^{18}O_{stem}(t) = \int_0^B \delta^{18}O(z) \cdot q(z,t)\,dz$$

??

$$q(z,t) = \frac{U(z,t)}{U_{Tot}(t)}$$

$$U_{Tot}(t) = \int_0^B U(z,t)\,dz$$

$$U(z,t) \propto RA(z) \cdot \ln\left[a \cdot RA(z)\right] \cdot \left[\Psi(z,t) \cdot k(z,t) - \Psi_{root}(t) \cdot k_{root}(t)\right]$$

$$RA(z) \equiv \omega \cdot Ga(\alpha_1, \mu_1) + (1-\omega) \cdot Ga(\alpha_2, \mu_2)$$

Soil respiration

$$\delta^{13}C_{Tot}(t) = \sum_{i=1}^{N_{source}} \left[\delta^{13}C_i \cdot \left(\int_0^B p_i(z,t)\,dz \right) \right]$$

??

$$p_i(z,t) = \frac{r_i(z,t) \cdot M_i(z,t)}{R_{Tot}(t)}$$

$$R_{Tot}(t) = \sum_{i=1}^{N_{source}} \left(\int_0^B r_i(z,t) \cdot M_i(z,t)\,dz \right)$$

$$r_i(z,t) = f\left(\theta_i, SWC(z,t), T(z,t)\right)$$

(Q_{10} Function, Energy of Activation)

$$M_i(z,t) \rightarrow known\ /\ measured\ ?$$

Isotope mixing model

Fractional contributions

Total flux

Flux model

Substrate or root profiles

The Data Models

Plant water uptake

What are
$\omega, \alpha_1, \mu_1, \alpha_2, \mu_2$**?**

$$RA(z) \equiv \omega \cdot Ga(\alpha_1, \mu_1) + (1-\omega) \cdot Ga(\alpha_2, \mu_2)$$
$$\Rightarrow U(z,t)$$
$$\Rightarrow q(z,t)$$

$$\begin{bmatrix} \delta D^{Obs}_{stem}(t) \\ \delta^{18}O^{Obs}_{stem}(t) \end{bmatrix} \sim MNo\left(\begin{bmatrix} \delta D_{stem}(t) \\ \delta^{18}O_{stem}(t) \end{bmatrix}, \Sigma \right)$$

Soil respiration

What is
θ_i**?**

$$r_i(z,t) = f\left(\theta_i, SWC(z,t), T(z,t)\right)$$
$$\Rightarrow R_{Tot}(t)$$
$$\Rightarrow p_i(z,t)$$

$$\delta^{13}C^{Obs}_{Tot}(t) \sim No\left(\delta^{13}C_{Tot}(t), \sigma^2_C\right)$$

$$R^{Obs}_{Tot}(t) \sim No\left(R_{Tot}(t), \sigma^2_R\right)$$

From isotope mixing model & flux model

The Inverse Solution

- **Bayesian estimation routine**
- **Markov chain Monte Carlo**
- **Plant water uptake**
  - **Metropolis-Hastings, Matlab**
- **Soil respiration**
  - **WinBUGS program**
- **Posterior distributions for:**
  - **Parameters:** ω, α_1, μ_1, α_2, μ_2, θ_i
  - **Fractional contributions:** $q(z,t)$, $p_i(z,t)$, $P_D(t)$, $P_S(t)$
  - **Profiles:** $RA(z)$, $U(z,t)$, $r_i(z,t)$