

Abstract: Let $u(x, t)$ be a (possibly weak) solution of the Navier - Stokes equations on all of \mathbf{R}^3 , or on the torus $\mathbf{R}^3/\mathbf{Z}^3$, with or without a divergence-free forcing term. The *energy spectrum* of $u(\cdot, t)$ is the spherical integral

$$E(\kappa, t) = \int_{|k|=\kappa} |\hat{u}(k, t)|^2 dS(k), \quad 0 \leq \kappa < \infty,$$

or alternatively, a suitable approximate sum. An argument involving scale invariance and dimensional analysis given by Kolmogorov in 1941, and subsequently refined by Obukov, predicts that in three dimensions, solutions of the Navier - Stokes equations at large Reynolds number and exhibiting fully developed turbulent behavior should obey

$$E(\kappa, t) \sim C\kappa^{-5/3},$$

at least in an average sense. I will explain a global estimate on weak solutions in the norm $|\mathcal{F}\partial_x u(\cdot, t)|_\infty$ which gives bounds on a solution's ability to satisfy the Kolmogorov law. The result gives rigorous upper and lower bounds on the inertial range, and in the unforced case an upper bound on the time of validity of the spectral regime. These results are joint work with Andrei Biryuk.